

Conservative discretizations and parameter-robust preconditioners for multiple-network poroelastic systems

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Multiple-network poroelastic theory (MPET) has been introduced in geomechanics to describe the mechanical deformation and fluid flow in media permeated by pores and fissures of different porosities and permeabilities as a generalization of Biot's theory, see [1]. More recently it has also been applied successfully in the modeling of cerebral water transport [4]. The parameters in the governing system of partial differential equations typically vary over several orders of magnitude making its stable discretization and efficient solution a challenging task, see [2, 3] for the case of Biot's consolidation model.

We generalize here our approach of using pointwise mass-conservative discretizations for the classical three-field formulation of the single-network poroelastic problem, cf. [2], to a flux-based formulation of multiple-network poroelastic systems. The key to establish the uniform inf-sup stability of the continuous problem is the choice of proper parameter-dependent norms. This allows also the parameter-robust transfer of the induced norm-equivalent preconditioners to the discrete level. We further prove the corresponding optimal error estimates.

REFERENCES

- [1] Bai, Mao and Elsworth, Derek and Roegiers, Jean-Claude, *Multiporosity/multipermeability approach to the simulation of naturally fractured reservoirs*, Water Resources Research **29**(6), pp. 1621–1633, 1993.
- [2] Q. Hong and J. Kraus, *Parameter-robust stability of classical three-field formulation of Biot's consolidation model*, arXiv:1706.00724v2 [math.NA], 2017. (Submitted).
- [3] Lee, Jeonghun J. and Mardal, Kent-Andre and Winther, Ragnar, *Parameter-robust discretization and preconditioning of Biot's consolidation model*, SIAM J. Sci. Comput. **39**(1), pp. A1–A24, 2017.
- [4] Tully, B. and Ventikos, Y., *Cerebral water transport using multiple-network poroelastic theory: application to normal pressure hydrocephalus*, Journal of Fluid Mechanics **667**, Cambridge University Press, pp. 188–215, 2011.