Kinetic Energy Preserving Discontinuous Galerkin Methods for Large Eddy Simulation

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In high order discontinuous Galerkin (DG) methods dissipation enters the scheme via the surface terms only. Hence, for very high order approximations, the effect of the dissipation changes and gets a more spectral cut-off like character: dissipation errors are very low for well resolved scales and are very high for scales close to the Nyquist cutoff. One can argue from this that dissipation in a very high order DG scheme acts like a high frequency filter, which is at first sight a desirable feature of the DG framework. Depending on the DG variant implemented, aliasing errors can affect the performance of the approximation and can even cause catastrophic blow ups in vortical driven flows. An ad hoc counter mechanism is so-called over integration, or polynomial de-aliasing. The first part of the talk shows that this implicit LES (iLES) strategy works well, as long as a substantial part of the dissipation is actually resolved. In our test case, the threshold seems to be about 40%. This threshold is not feasible for industry applications. Unfortunately, our investigations show that there is no obvious way to improve the result for coarser resolutions, as adding a turbulence model does not increase the accuracy at all. Thus, in the second part, we introduce a novel strategy to model under-resolved turbulence within the DG framework. Recent Split Form DG methods\(^1\) include kinetic energy preserving variants, which allow for a relative precise control of the discrete kinetic energy dissipation, while having inbuilt de-aliasing and hence nice robustness properties. We start with a kinetic energy preserving DG method as a baseline scheme and carefully add subgrid scale modelling. Our investigations show that within this novel LES DG approach, the turbulence model actually matters and that it is possible to substantially improve the results compared to the standard iLES approach\(^2\).

REFERENCES
